Lesson 16 (3.8)

Today: Implicit Differentiation

Office Hours: MWF: 2:45PM – 4:15PM, MATH 842.

Announcements:

* Exam 2: Wednesday, Oct 15th 8PM – 9PM
* Final Exam: Monday, Dec 15th 1PM – 3PM

instructions on Brightspace.

* HW 16, 17 : Due on Tuesday
* Quiz 10 (Lesson 15) : On Tuesday

Warmup example:

$y = (f(x))^3$ find y'

eg:

$y = (\sin x)^3$, $u = \sin x$

$y' = \frac{du^3}{du} \cdot \frac{du}{dx} = 3(\sin x)^2 \cdot \cos x$

$y = [f(x)]^3$, $u = f(x)$

$y' = \frac{du^3}{du} \cdot \frac{du}{dx} = 3(f(x))^2 \cdot f'(x)$

Explicit function v/s Implicit function

Most functions that we have seen so far

eg: $y = 7x - 5$

$y = \sin x$

$y = e^{\sin x}$

$y = \dfrac{\tan(x^3+5)}{e^{\sin x}}$

$y = f(x)$

dependent variable

independent variable

- Compute Derivatives using the Rules learnt

* y is still dependent on x but you donot have a formula directly given

* y is not explicity written interm of only y.

Find y' using implicit Diff.

→ $x^2 + y^3 = 5$

↳ I can actually for y here

$y = (5 - x^2)^{1/3}$

* $x^2 + xy - y^3 = 7$

— May be we could write $y = f(x)$ but not easy.

* $\sin(xy) = x^2 + y$

— Very Very Hard to write $y = f(x)$

eg) $x^2+y^3=5$, find $\frac{dy}{dx}=y'$

Implicit Differentiation!

① take derivative w.r.t x on both sides keeping in mind that y here is actually $y(x)$

$$\frac{d}{dx}\left(x^2+y^3\right)=\frac{d}{dx}5$$

$$2x+\boxed{\frac{d}{dx}(y(x))^3}=0 \rightsquigarrow 2x+3y^2y'=0$$

$=3y^2\cdot y'$ (using chain Rule)

② Solve for y' in terms of x & y

$$2x+3y^2y'=0 \rightsquigarrow \boxed{y'=\frac{-2x}{3y^2}}$$

Verify implicit Differentiation!

$x^2 + y^3 = 5 \rightsquigarrow y = (5-x^2)^{1/3}$

$$y' = \frac{d}{dx}(5-x^2)^{1/3}$$

$$= \frac{1}{3}(5-x^2)^{-2/3} \cdot (-2x)$$

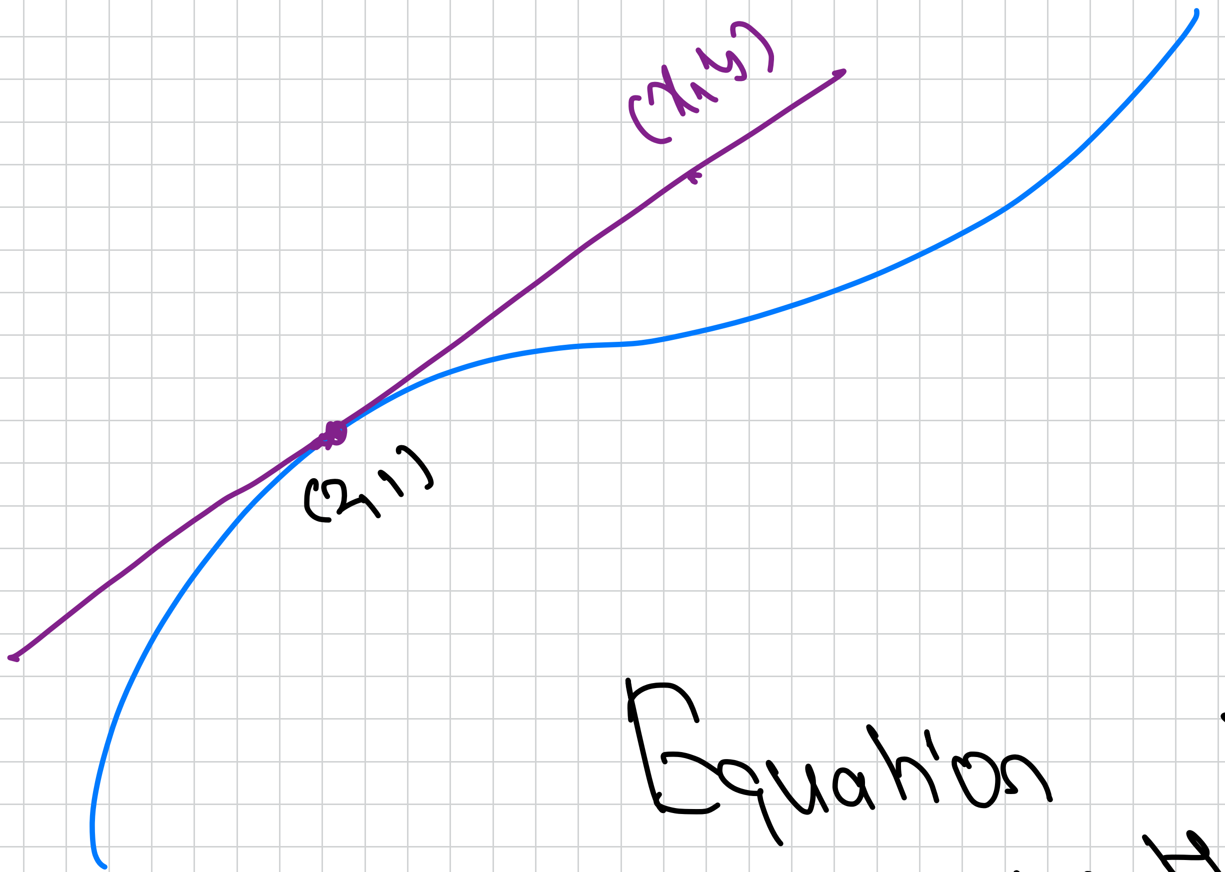
$$= \frac{-2x}{3\left((5-x^2)^{1/3}\right)^2}$$

$$= \frac{-2x}{3y^2}$$

find equation of the tangent line to $x^2+y^3=5$ at $(2,1)$.

$\Downarrow$

slope of the tangent line at $x=2,\ y=1$



$$y' = \frac{-2x}{3y^2} \rightsquigarrow y'\Big|_{(2,1)} = \frac{-2(2)}{3(1)^2} = -4/3.$$

Equation of Line with slope $-4/3$, passing through $(2,1)$

point slope form: $\frac{y-1}{x-2} = -4/3 \rightsquigarrow y-1 = \frac{-4}{3}x + \frac{8}{3}$

$$\boxed{y = -\frac{4}{3}x + \frac{11}{3}}$$

e.g. $e^y = x\sin y$, find y'

① $\frac{d}{dx}$ on Both sides

$$\frac{d}{dx}\left[e^{y(x)}\right] = \frac{d}{dx}\left[x \cdot \sin(y(x))\right]$$

chain Rule

product Rule

$$e^{y(x)} \cdot y' = \left(\frac{d}{dx}x\right)\left(\sin(y(x))\right) + x\,\frac{d}{dx}\left[\sin(y(x))\right]$$

chain Rule

$$e^{y} \cdot y' = \sin y + x\left[(\cos y) \cdot y'\right]$$

② Solve for y'

$$e^y \cdot y' - x\cos y \cdot y' = \sin y$$

$$y'\left[e^y - x\cos y\right] = \sin y \rightsquigarrow \boxed{y' = \frac{\sin y}{e^y - x\cos y}}$$

e.g.: $\sin(xy) = x^2 + y$, find y'

① $\frac{d}{dx}$ on Both sides

$$\frac{d}{dx}\left[\sin(\underset{u}{xy})\right] = \frac{d}{dx}\left[x^2 + y\right]$$

$$\frac{d}{du}\sin u \cdot \frac{du}{dx} = 2x + y'$$

$$\cos u \cdot \frac{d}{dx}(xy) = 2x + y'$$

$$\cos(xy)\left[\frac{d}{dx}x \cdot y + x\frac{dy}{dx}\right] = 2x + y' \rightsquigarrow \cos xy\,(y + xy') = 2x + y'$$

② Solve for y'

$$y\cos(xy) + x\cos(xy) \cdot y' = 2x + y'$$

$$y' = \frac{2x - y\cos(xy)}{\left[x\cos(xy) - 1\right]}$$

eg: $x^2 + xy - y^3 = 7$, find y'' $\rightsquigarrow$ $y'' = \frac{d}{dx} y'$

① $\frac{d}{dx}$ on Both Sides

$$\frac{d}{dx}\left[x^2 + xy - y^3\right] = \frac{d}{dx} 7$$

$$2x + \left[\frac{dx}{dx} \cdot y + x \frac{dy}{dx}\right] - 3y^2 \frac{dy}{dx} = 0$$

$$2x + y + xy' - 3y^2 y' = 0$$

$$\left[\begin{aligned} &2x + y + xy' - 3y^2 y' = 0 \\ &y' = -\frac{(2x+y)}{x - 3y^2} \end{aligned}\right.$$

② $\frac{d}{dx}$ again on Both sides

$$\frac{d}{dx}\left[2x + y + xy' - 3y^2 y'\right] = 0$$

$$2 + y' + \left(\overset{1}{\frac{d}{dx} x} \cdot y' + x \cdot \overset{y''}{\frac{d}{dx} y'}\right) - 3\left[\left(\frac{d}{dx} y^2\right) y' + y^2 \overset{y''}{\frac{d}{dx} y'}\right] = 0$$

Chain Rule $\rightarrow$ $2yy'$

$$2 + y' + \left(\underbrace{\frac{d}{dx}x}_{1} \cdot y' + x \cdot \underbrace{\frac{d}{dx}y'}_{y''}\right) - 3\left[\underbrace{\left(\frac{d}{dx}y^2\right)}_{2yy'} y' + y^2 \underbrace{\frac{d}{dx}y'}_{y''}\right] = 0$$

Chain Rule $2yy'$

$$2 + y' + y' + xy'' - 3\left[2y(y')^2 + y^2 y''\right] = 0$$

$$\left[2 + 2y' - 6y(y')^2\right] + y''\left[x - 3y^2\right] = 0$$

$$y'' = -\frac{\left(2 + 2y' - 6y(y')^2\right)}{\left(x - 3y^2\right)}, \quad y' = \frac{-(2x+y)}{x - 3y^2}.$$